

<b>INTERMAGNET Technical Note</b>		
<b>Title:</b> Testing the timing accuracy of 1s INTERMAGNET variometer.		
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<b>Purpose of document:</b> Describe a black-box testing procedure on a 1s INTERMAGNET variometric data acquisition chain for ascertaining its timing accuracy.		
<b>Terms of Reference</b>		
<ol style="list-style-type: none"> <li>1. The Technical note describes a black-box testing procedure on a 1s INTERMAGNET variometric data acquisition chain for ascertaining its timing accuracy.</li> <li>2. The specialized equipment necessary for performing the test is described</li> <li>3. The data processing used to derive the tests result is described</li> </ol>		
<b>Date due:</b>		<b>Date submitted:</b>
<b>Outcome:</b> <i>(to be completed by the Secretary of the Operations Committee.)</i>		
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## 1. Introduction

At the Dourbes OPSCOM/EXCON meeting in 2003, a decision was made to launch a new standard for recording variometric data within INTERMAGNET with a sampling rate of 1 Hertz.

A user survey (Love 2004) on 1 second [s] variometric data needs established that 10 milliseconds [ms] is the general consensus on timing accuracy for 1s sampling. The procedure addressed in this note relates to the seeming difficulty to establish a timing accuracy two orders of magnitude higher than the sampling rate.

It is considered important that an IMO be able to test the timing accuracy of its geomagnetic field observations and not solely rely on the manufacturer claims.

INTERMAGNET envisions helping IMO's with the testing of their prospective 1s variometric systems by dispensing advice, lending test equipment and making available analysis software.

## 2. Background

- 1) The 1s INTERMAGNET variometer is a triaxial magnetometer providing measurements of the variations of the geomagnetic field vector at a sample rate of 1 Hz.
- 2) The overall timing accuracy with respect to UTC of the 1s INTERMAGNET variometer is requested to be not worse than 10 ms (minutes of OPSCOM/EXCON Mexico meeting 2006)
- 3) A band-limited measurement cannot provide data without introducing a phase-lag. A 1s variometer is necessarily band-limited and therefore the accuracy mentioned in 2) above is in question. However, the procedure of sampling allows shifting the timestamp on the data. Therefore if the analogue and digital part of the variometer use filters of the linear phase type, and the resulting constant delay is a multiple of the sampling rate, an adequate timestamp shift can theoretically result in a data acquisition with the required timing accuracy.
- 4) The 1s INTERMAGNET sample must refer to the top of the UTC second, i.e. the epoch hh:mm:ss.000 (minutes of OPSCOM/EXCON Mexico meeting 2006)
- 5) All components measured by the 1s INTERMAGNET variometer must be sampled synchronously so that the overall timing accuracy is maintained
- 6) For a periodic signal of period T, the relationship between timing t and phase lag  $\Phi$  in degrees is given by:  $t = T \times (\Phi/360)$

## 3. Practical Requirements resulting from IMO constraints

- 1) The testing should be executable on any geomagnetic variometer
- 2) The testing procedure should be simple enough to be carried out in a magnetic observatory modestly equipped with testing equipment. A voltmeter and oscilloscope may be necessary
- 3) The testing procedure describes the design of a specialized device needed for the test. INTERMAGNET intends to make it available to IMOs for their testing purposes
- 4) For performing the timing test, there should be no necessity to modify the variometer installation, nor to dismantle nor stop the magnetometer electronics or data acquisition. Black-box testing should be performed by injecting a magnetic signal to the variometer sensor and analysing the resulting output 1s INTERMAGNET file

## 4. Comments on current practices

- i) Up to now, concerning the standard sampling of minute means, INTERMAGNET has not required testing of magnetometers for ascertaining their timing accuracy.

- ii) The only mention of timing accuracy in time series for data acquisition in the INTERMAGNET literature (technical manual) has been the limitation of clock drift to 5 s
- iii) Testing of INTERMAGNET variometers has mainly regarded their long-term stability affected by temperature coefficients and mechanical changes in their geometry. This testing is carried out routinely by baseline checks using absolute measurements or by direct intercomparisons between two or more instruments, one of them possibly subjected to increased levels of perturbations

## 5. Black-box testing procedure

### 5.1 A black-box device: the 1s INTERMAGNET variometer and data acquisition

- i) The black-box is a device which is accessible only at its input and output. There is no possibility to discover its properties or derive its characteristics otherwise by e.g. peering into it or dismantling it.
- ii) In this technical note, the black-box is the 1s INTERMAGNET variometer plus its data acquisition system, considered as a single device. The input is therefore the magnetic field surrounding the variometer sensor and the output is the computer file with the magnetic data record sampled at 1 Hz.
- iii) The input will be provided in part by a coil system sitting close to or around the variometer sensor and producing a magnetic field there. This field will be sensed by the variometer in addition to the geomagnetic field, following the superposition principle.

### 5.2 The test signal input to the black box.

- i) The test signal  $S(t)$  will be a periodically variable magnetic field of constant period  $T$  (frequency  $f=1/T$ ) and amplitude  $A$ .
- ii) The value of the signal period  $T$  is taken well within the pass-band of the variometer.
- iii) The phase  $\Phi$  of this test signal will be referred to an UTC epoch  $t_0 = \text{hh:mm:ss.000}$  fixed beforehand in each testing experiment. The phase must be a constant, ideally respecting the condition  $\Phi=0$ .
- iv) Timing of this test signal  $S(t)$  should be at least an order of magnitude better than the 0.01s variometer requirement. Therefore  $t_0$  and the phase shift requirement in 6) of §2 above should be known with an error not worse than 0.001 s.
- v) The waveform of the test signal may be a sine, triangle or square wave. It is expected that generating this test signal will be easier in the form of a triangle or square wave.
- vi) In order to respect the accuracy of the timing requirements it is recommended that the test signal be directly derived from a GNSS or UTC related terrestrial broadcasted signal.
- vii) In order to maximize the signal-to-noise ratio, the amplitude of the test signal should be made as big as possible. But care should be exercised so as to avoid saturation of the variometer. A pk-pk amplitude of 4000 nT should be satisfactory in this respect. Perturbation inflicted on nearby geomagnetic equipment should also be considered when determining the maximum amplitude of  $S(t)$ .
- viii) The real thing to worry about is to have the current in the coil with no delay or phase lags, and this should not be a problem at the millisecond level. Anyway it can be monitored with an oscilloscope measuring the voltage across a resistor in series with the coil. In case of triangle signal, a high linearity should be obtained. It is recommended that non-square wave test signals be tested in the same way as the recorded signal in order to establish any timing inaccuracies of their fundamental (first harmonic).

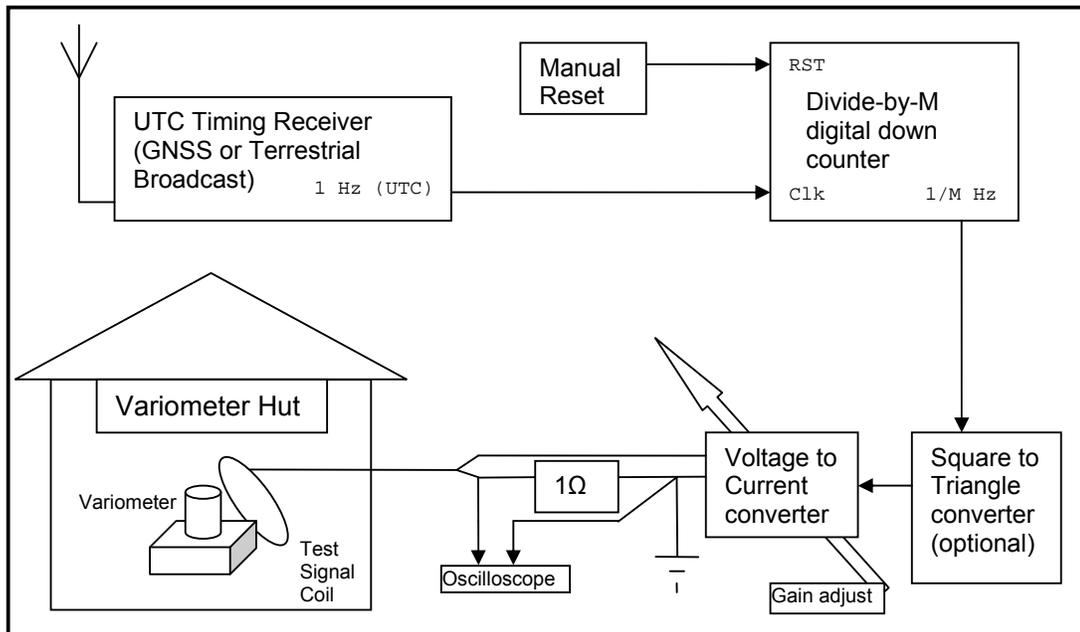
### 5.3 Proposed experiment

A timing receiver (Figure 1) produces 1 Hz pulses synchronized to UTC. They increment a symmetrical<sup>1</sup> digital divide-by-M down counter providing an output pulse train at the exact frequency 1/M Hz.

A manual reset switch is operated until a while before  $t_0$  so that the rising edge of the following UTC 1 Hz pulse will change the output state of the counter. Therefore the counter output will show a rising edge at  $t_0$ . The next output rising edge will occur M seconds thereafter and will repeat with that pattern. This counter output signal represents a logic signal of symmetrically alternating highs and lows. The epoch  $t_0$  is thus fixed. It is useful to have access to a reference clock when operating the manual Reset switch and write down the observed value of  $t_0$ .

This signal is then transformed into a triangular wave by an electronic device providing linearly increasing voltages during highs and decreasing voltages during lows of the logic signal. A known way to do this is by charging or discharging a capacitor at constant current.

It can also be decided to run the test with a square wave signal. In that case the triangle converter is bypassed.



**Figure 1. Layout of the proposed timing experiment**

The signal is then converted to a current with a conversion factor of K Amperes/Volt by a voltage-to-current amplifier. The value of K can be adjusted by a control “Gain adjust” in order to maximize the signal-to-noise ratio of the test. The resulting current is then sent over a bifilar cable to the coil sitting close or around the variometer under test.

In the return line of the cable, a 1 ohm resistor is inserted so that the current in the coil can be unobtrusively measured by inspecting the voltage across it. This can easily be done by an oscilloscope.

### 5.4 The variometric recording of the test signal

- i) The recording will start with the sample at  $t_0 = \text{hh:mm:ss.000}$
- ii) The recording will last for a total of N samples, making the test duration equal to N seconds.
- iii) The recording will be in the usual 1s INTERMAGNET format

<sup>1</sup> Symmetrical counter means that the output pulse train will have 50% duty cycle

### 5.5 Analysis of the test record signal

The analysis is based on the idea of linear least squares parameter estimation on a signal having a known frequency. The idea comes from Earth Tidal analysis techniques where the amplitude and phase estimation of periodic signals at the lunisolar frequencies can be performed with high accuracy, even with low sampling rates and in presence of noise, provided that a long enough series of data is available.

A curve having the form  $s(t) = a + b \cos\left(\frac{2\pi}{T}t + \varphi\right)$  is fitted in the least squares sense to

the recorded data and  $a$  (offset),  $b$  (amplitude) and  $\varphi$  (phase lag) are the parameters to estimate in order to minimise the variance. It can be shown (Malacara 1998) that the least squares estimation is linear if  $T$  is known, making the method robust and accurate in all cases:  $a$ ,  $b$  and  $\varphi$  can be calculated by a simple formula encompassing the  $N$  samples record as well as the values  $N$  and  $T$ . Details of the computations are given below and illustrated in an excel file attached to this document.

As the sinusoidal least squares estimation concerns only the first harmonic of the recorded test signal, it is not necessary to inject a pure sine wave as test signal. Indeed any periodic signal would do as long as its period is constant and it's fundamental has  $\Phi=0$  or known with sufficient accuracy. Therefore preference is given to square or triangular waves, which are quite easy to generate with these characteristics.

The timing error  $E$  is then obtained as  $E = T^*(\varphi/360)$ . The other parameters  $a$  and  $b$  are not really useful in this context.

Example: Given a test signal period at  $T = 16s$ , if  $\varphi$  is found to be  $2.0^\circ$  by the least squares computation, then  $E = 16*(2/360) = 0.089s$

Of course the natural variations of the field will interfere with the estimation, but as noted above, the amplitude  $b$  can be made big enough and the series long enough with numerous cycles in the test signal until the effect of these variations become negligible by statistical averaging.

The test can be repeated at various frequencies, in order to establish the variometer phase lag dependency on frequency.

### 5.6 Formulation for least squares parameter estimation on a sinusoidal signal

The sinusoidal signal curve fit

$$s(t) = a + b \cos\left(\frac{2\pi}{T}t + \varphi\right)$$

can be rewritten as

$$s(t) = D_1 + D_2 \cos\frac{2\pi}{T}t + D_3 \sin\frac{2\pi}{T}t$$

where

$$D_1 = a$$

$$D_2 = b \cos \varphi$$

$$D_3 = -b \sin \varphi.$$

The data record of the test signal will contain the  $N$  samples:

$$s_n = D_1 + D_2 \cos\frac{2\pi}{T}t_n + D_3 \sin\frac{2\pi}{T}t_n, \quad n = 0, \dots, N-1 \quad N \geq 3$$

The best fit in the least squares sense is obtained if the  $D$  parameters  $D_1$ ,  $D_2$  and  $D_3$  are chosen so as to minimize the variance  $\varepsilon$ , defined by

$$\varepsilon = \frac{1}{N} \sum_{n=0}^{N-1} (D_1 + D_2 \cos \frac{2\pi}{T} t_n + D_3 \sin \frac{2\pi}{T} t_n - s_n)^2$$

Taking the partial derivatives of  $\varepsilon$  with respect to the D parameters leads to the matrix equation:

$$\begin{pmatrix} N & \sum \cos \frac{2\pi}{T} t_n & \sum \sin \frac{2\pi}{T} t_n \\ \sum \cos \frac{2\pi}{T} t_n & \sum \cos^2 \frac{2\pi}{T} t_n & \sum \cos \frac{2\pi}{T} t_n \sin \frac{2\pi}{T} t_n \\ \sum \sin \frac{2\pi}{T} t_n & \sum \cos \frac{2\pi}{T} t_n \sin \frac{2\pi}{T} t_n & \sum \sin^2 \frac{2\pi}{T} t_n \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} \sum s_n \\ \sum s_n \cos \frac{2\pi}{T} t_n \\ \sum s_n \sin \frac{2\pi}{T} t_n \end{pmatrix}$$

This general equation can be solved for the D parameters by inversion of the symmetric 3x3 matrix. Note this matrix depends only on the test signal frequency  $1/T$ , the number of samples  $N$  and the sampling rate, but does not depend on the measured signal  $s_n$ . It can be computed once and for all for a given experiment set-up.

Also the procedure does not call for a constant sampling rate; therefore gaps in a recording – if any – can be accommodated.

The final result is obtained as:

$$\tan \varphi = -\left( \frac{D_3}{D_2} \right),$$

The value of  $\varphi$ , extracted from this equation, must be corrected by the amount  $\Phi$  if it is different from zero.

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